Preventing Bad Plans by Bounding the Impact of Cardinality Estimation Errors

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August 27, 2009
Motivation

Query optimization, in particular join ordering, has a large impact.

- optimization relies upon a cost model
- a central component: *cardinality/selectivity estimation*
- usually relies upon statistical synopses
Motivation (2)

Estimation errors can have very unfortunate consequences.

\[ R = \text{real cardinalities}, \ E = \text{estimated cardinalities} \]

Plan A

\[
\begin{align*}
\sigma(R_1) \quad & R 1000 \\
& E 200 \\
\sigma(R_3) \quad & R 10,000 \\
& E 5,000 \\
\sigma(R_3) \quad & R 400 \\
& E 1000 \\
R_2 \quad & R 100,000 \\
& E 100,000 \\
\end{align*}
\]

\[ R 100,000 \text{ intermediate tuples} \]
\[ E 20,000 \text{ intermediate tuples} \]

\begin{itemize}
  \item errors amplify each other
  \item errors propagate multiplicatively
\end{itemize}

Plan B

\[
\begin{align*}
\sigma(R_1) \quad & R 1000 \\
& E 200 \\
\sigma(R_3) \quad & R 10,000 \\
& E 5,000 \\
\sigma(R_3) \quad & R 400 \\
& E 1000 \\
R_2 \quad & R 100,000 \\
& E 100,000 \\
\end{align*}
\]

\[ R 10,000 \text{ intermediate tuples} \]
\[ E 25,000 \text{ intermediate tuples} \]

How can we limit the effect of estimation errors?
Motivation (3)

Misestimations can be even more painful with physical cost models:

- $R_1$ (without $\sigma$), could be large, e.g. $|R_1| = 10^7$
- then such estimation errors can easily happen for mildly skewed data

We observed such numbers in a commercial database system!

- estimates get more and more ”dangerous” the smaller they get
- but overestimations are just as bad
- must take the effect on plans into account
Overview

1. Motivation
2. **Bounding the Impact of Cardinality Estimation Errors**
3. Constructing Synopses
4. Evaluation
5. Conclusion
Measuring Estimation Errors

When estimating cardinalities **error are inevitable** in general
  - synopsis usually try to minimize errors
  - different error metrics to choose from

For simplicity, consider point queries of the form $\sigma_{a=\text{const}}(R)$
  - let $f(x)$ be $|\sigma_{a=x}(R)|$ for $x \in D_a = \Pi_a(R)$
  - let $\hat{f}(x)$ be the estimated derived from a synopsis

Popular error metrics (＝ optimization goals)

\[
\begin{align*}
  l_2 &= \sqrt{\sum_{x \in D_a} (f(x) - \hat{f}(x))^2} \\
  l_\infty &= \max_{x \in D_a} |f(x) - \hat{f}(x)|
\end{align*}
\]

Minimizing these error can lead to **arbitrarily bad plans!**
Defining the Q(uotient) Error

- as errors propagate multiplicatively, the metric should be multiplicative
- it should be symmetric regarding over- and underestimation

We define the $q$-error as follows

$$l_q = \max_{x \in D_a} \frac{\max(f(x), \hat{f}(x))}{\min(f(x), \hat{f}(x))}$$

(we also use the notation $||\hat{f}/f||_Q$ for $l_q$)

- true cardinality 10, estimation 100 $\Rightarrow l_q = 10$
- true cardinality 10, estimation 1 $\Rightarrow l_q = 10$
- note that $l_q$ is the maximum over the whole domain

Knowing the $q$-error allows for deriving **bounds on the resulting plan**! 
Optimality of the Resulting Plan

Notation: Using the estimates \( \hat{f} \) instead of the true cardinalities \( f \) transforms a cost function \( C \) into \( \hat{C} \).
\( f_i \) is the selectivity of \( \sigma_{p_i}(R_i) \), \( f_{i,j} \) the join selectivity \( R_i \Join R_j \).

**Theorem:** Let \( C \) be a cost function with ASI property. For a given chain query in \( n \) relations, let \( P \) be the optimal left-deep plan without cross products under \( C \), and \( \hat{P} \) be the optimal left-deep plan without cross products under \( \hat{C} \). If for all \( 1 \leq k \leq n \)

\[
\frac{||\hat{f}_k||_Q}{f_k} < \min_{i \neq j-1} \sqrt{\frac{f_i f_{i,i+1} |R_i|}{f_j f_{j,j-1} |R_j|}} ||_Q =: q,
\]

then \( C(\hat{P}) = C(P) \).

I.e., the optimizer will find the **real optimal solution** by using the estimates if the \( l_q \) error is bound by \( q \).
Optimality of the Resulting Plan (2)

We could show similar optimality bounds for
- star queries
- tree queries
- star queries using sort-merge joins (non-ASI cost function)

In these cases we could show optimality if $l_q$ is bound by a suitable value $q$.

- for grace-hash-join not proof yet
- but a numerical study suggests the same

We cannot always meet bounds in practice (e.g., for ties in selectivity), but first metric with proofs of optimality deriving from estimation errors.
Theorem: Let $C = C_{SMJ}$ or $C = C_{GHJ}$. For a given query in $n$ relations, let $P$ be the optimal plan under $C$, and $\hat{P}$ be the optimal plan under $\hat{C}$. Then

$$C(\hat{P}) \leq q^4 C(P),$$

where $q$ is defined as

$$q = \max_{x \subseteq X} \left\| \frac{\hat{S}_x}{S_x} \right\|_Q,$$

with $X$ being the set of relations to be joined. That is, $q$ is the maximum estimation error taken over all intermediate results.
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What kind of Synopsis?

- construct synopsis that minimize the $q$-error
- our algorithm can construct bucket histograms with polynomials

$f(x)$

degree 0 $f(x) = c$ is very common
degree 1 $f(x) = mx + c$ is often more effective (with given space)
degree $\geq 2$ does not pay off most of the time

We concentrate on line segments here.
Algorithm operates in two layers:
1. single bucket construction
   - fits a line to given data points
2. multi-bucket constructions
   - chooses the bucket boundaries
   - uses single-bucket construction as building block

Algorithm constructs the optimal fit for the $q$-error
- math is somewhat involved and lengthy
- approach is even more general than just polynomials
- formal proof of optimality is in the paper
- will only sketch the idea for linear fits here
Single-Bucket Construction

We fit a function $\alpha x + \beta$ to data points $x_1, \ldots, x_n$

1. $n = 1$ (trivial)
   \[ \alpha = 0, \beta = f(x_1) \]

2. $n = 2$ (trivial)
   \[ \alpha = \frac{f(x_2) - f(x_1)}{x_2 - x_1}, \beta = f(x_1) - \alpha x_1 \]

3. $n = 3$ (solve equation system)

   \[
   \alpha = (x_3 - x_1) \sqrt{\frac{f(x_2)}{(x_3 - x_1)[f(x_3)(x_3 - x_1) + (f(x_3) - f(x_1))(x_3 - x_2)]}}
   \]

   \[
   \beta = \alpha f(x_3) \frac{x_3 - x_1}{f(x_3) - f(x_1)} - \alpha x_3 \text{ if } \alpha \neq 0
   \]

   \[
   = f(x_1) \sqrt{\frac{f(x_2)}{f(x_1)}} \text{ if } \alpha = 0
   \]
For $n > 3$ we use an iterative algorithm:

1. pick three data points, compute a linear fit
   - gives a lower bond for the estimation error
2. find the data point with the maximum estimation error
   - we can potentially improve by fitting to it
3. chose it as base point, replaces one of the three
4. repeat until the maximum error remains the same

Algorithm in the paper converges faster but is more complex.

Iterative approach, very fast in practice.
Multi-Bucket Construction

We place bucket boundaries by using the single-bucket construction

- the first bucket start is $x_1$
- the estimation error grows monotonic with the bucket size
- use binary search to find the maximum size such that $l_q \leq \epsilon$
- construct additional buckets until domain is covered
- perform binary search over $\epsilon$ until space budget is met

$$f(x)$$
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Evaluation

We compared different approaches:

- piecewise LinQ (our approach)
- V-Optimal histograms
- Wavelets
- Sampling
- Koenig histograms (piecewise linear, fits $l_2$)
- equi-depth histograms

All were given the same space budget

Data set: CDF of TF*IDF*PageRank scores from TREC-12

- 427,940 data points
- more experiments in the paper
Fit with a given Space Budget

Approximation error of other approaches varies greatly over the domain.
Q-Errors of Different Approaches

Note the logarithmic scale and that the plan costs are bound by $q^4$!
Conclusion

For cardinality estimation $l_q$ is a much more meaningful metric

- if the $q$-error can be bound suitably **optimality is guaranteed**
- if the bounds cannot be met, the **cost degradation is bound** by $q^4$

We developed a synopsis construction algorithm for minimizing $l_q$

- fast iterative algorithm for single-bucket construction
- can fit a wide class of functions
- efficient multi-bucket construction using binary search
- provides much better approximations than other approaches
- in particular, more **robust**, plan quality is bound!